NEW SOUTH WALES

Department of Education

LEAVING CERTIFICATE EXAMINATION, 1962

Mathematics II

PASS PAPER

Chief Examiner: T. G. ROOM, Sc.D.

Assessors: H. MULHALL, B.Sc., Ph.D.
G. C. FRANKS, B.Sc., Dip.Ed.

Time allowed—Three hours

Candidates may attempt all questions.

Question 1 carries 30 marks and the other questions 10 marks each.

Except in Question 1, marks will not be awarded to answers where the work is not shown.

Marks will be deducted for careless or badly arranged work.

Mathematical Tables and Squared Paper will be provided.

Slide rules may not be used.

Answers to the two Parts of this Paper are to be returned in *separate books* marked A or B.

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Part A

1. (i) In the triangle ABC, $a = 5$ units, $b = 7$ units, $\cos C = \frac{3}{7}$. Find as surds in their simplest forms

(a) the length of the side c;

(b) the area of the triangle.

(ii) A is the point (1,3) and B is the point (7, —1). Find

(a) the coordinates of the mid-point of AB;

(b) the equation of the right bisector of AB.

(iii) Show that (1,1), (4,2) and (6, —4) are the vertices of a right-angled triangle.

(iv) Find as a surd, without using tables, the value of the product $\sin 52\frac{1}{2}° \sin 7\frac{1}{2}°$.

(v) Find in radians all the values of x in the range $0 \leqslant x \leqslant \pi$ which satisfy the equation

$$\sin 2x = \sin x .$$

(vi) Given that

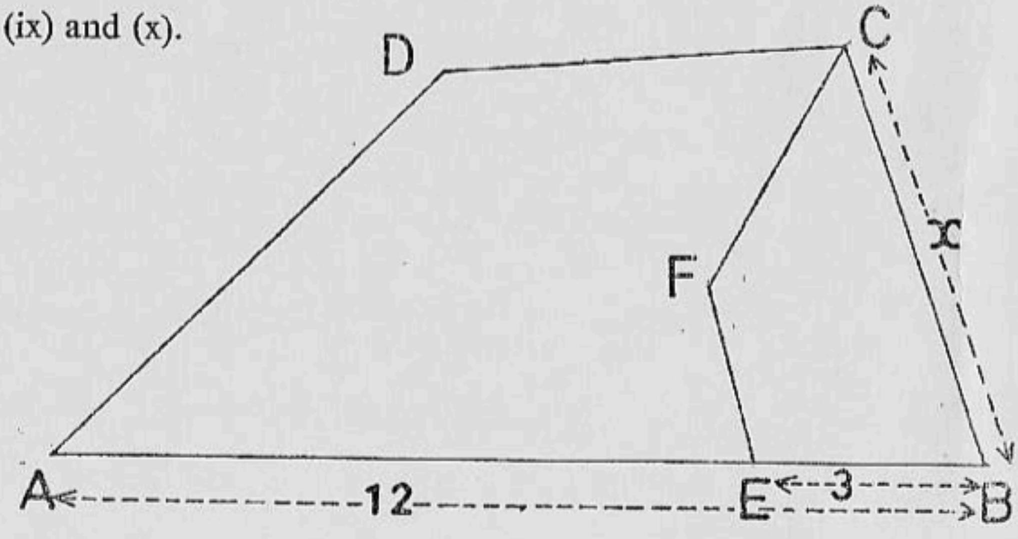
$$3 \cos (x + 45°) = \cos (x - 45°)$$

find $\tan x$.

(vii) Write down the equation of the tangent to the parabola $x = at^2$, $y = 2at$ at the point $t = 3$.

(viii) The focus of a parabola is the point (0, —4) and the directrix is the line $y = 4$. Write down the equation of the parabola and draw a rough sketch.

(ix) and (x).



(ix) In the given figure the quadrilaterals ABCD, CBEF are similar (with C, B, E, F corresponding respectively to A, B, C, D). If AB = 12 units, EB = 3 units, BC = x units, find x.

(x) If the area of ABCD is 36 square units, find the area of the polygon AEFCD.

2. ABC is a triangle; A is the point (0, 4), B is the point (—5, —1) and the orthocentre is the origin. Find

(*a*) the coordinates of C;

(*b*) the area of the triangle;

(*c*) the angle BAC.

3. D, D′ are the fixed points $(d,0)$, $(-d,0)$ and P is a variable point. Find the equations of, and describe geometrically, the loci of the points P which satisfy the conditions

(*a*) $PD = PD'$;

(*b*) $PD^2 + PD'^2 = 4d^2$;

(*c*) $\tan PDD' + \tan PD'D = 1$.

Part B

4. (i) Prove that

$$\frac{1 - \cos \alpha}{1 + \cos \alpha} = \tan^2 \tfrac{1}{2}\alpha .$$

By using this formula or otherwise find tan 15° as a surd in its simplest form.

(ii) If

$$\cos \alpha = \tan x \cot y,$$

show that

$$\tan^2 \tfrac{1}{2}\alpha = \sin(y - x) \operatorname{cosec}(y + x) .$$

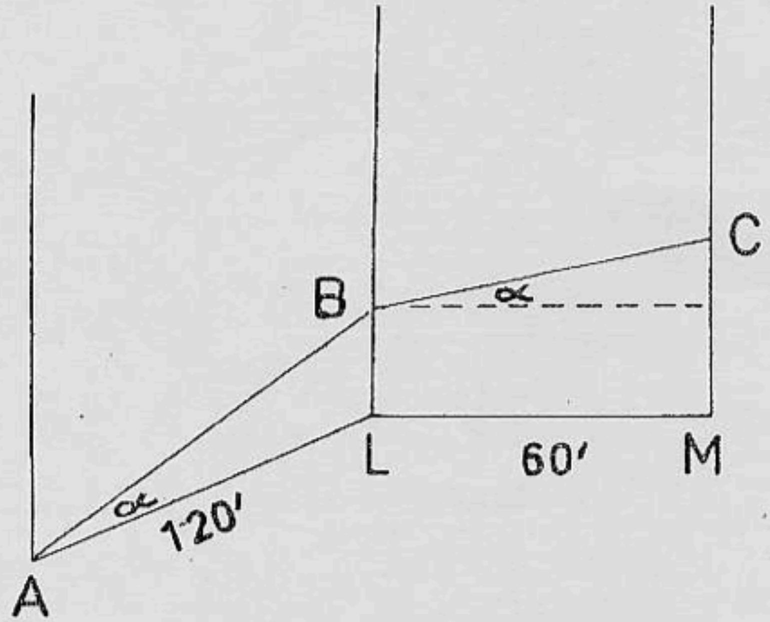
5. Find in degrees the values of x between 0° and 360° which satisfy the equation

$$\cot x = \tfrac{2}{3} \sin x .$$

Sketch (not on graph paper) on the same diagram the graphs of the functions $\cot x$ and $\frac{2}{3} \sin x$ for values of x in the range 0° to 360°, and state the ranges of values (between 0° and 360°) for which

$$2 \sin x > 3 \cot x.$$

6.



The diagram above represents part of the inside of two of the walls (built vertically and at right angles) of a parking station. In this diagram ALM represents the (horizontal) ground floor, and the lines AB and BC represent the ramp climbing up the two walls at a constant angle α, where $\tan \alpha = \frac{1}{6}$.

If AL = 120 feet and LM = 60 feet, find

(*a*) the length of AC to the nearest foot;

(*b*) the angle ABC to the nearest minute.

7. A, B are the points of contact of the tangents from P, (x_0,y_0), to the circle $x^2 + y^2 = a^2$.

Find the equation of AB.

A point Q lies on AB; C, D are the points of contact of the tangents from Q to the circle. Prove that

(*a*) P lies on CD;

(*b*) $PA^2 + QC^2 = PQ^2$.

8. The points P_1 and P_2 on the parabola $x = at^2$, $y = 2at$ have parameters t_1 and t_2 respectively. Find

(*a*) the equation of the normal at P_1;

(*b*) the coordinates of the point of intersection, Q, of the normals at P_1 and P_2;

(*c*) the equation of the chord P_1P_2.

If the chord P_1P_2 of the parabola passes through the point $(-2a,0)$, prove that $t_1t_2 = 2$ and that the point Q lies on the parabola.