Last Lecture, 2010
Mathematics: Pattern & Structure
The Human Condition: Free Will & Consciousness

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First Theorem of the day

Theorem

There is no fair voting system

Proof.

Later □

Outline

1. What is Mathematics?
2. The Physical Universe
   - Voting Systems (it ain't fair) and Social Choice
   - Maximising Satisfaction in an Economic System
   - Fractals, Money Markets, Disaggregated Materials
   - Factors, Code breaking and Internet Security
   - Google and Searching
3. The Human Condition
   - Free Will
   - Consciousness
4. Reflections on Teaching
5. Wrap Up

Mathematical Perspectives

Life is good for only two things, discovering mathematics and teaching mathematics.
Siméon Poisson, 1781–1840

(I disagree: JH)

Mathematics takes us ... into the region of absolute necessity, to which not only the actual world, but every possible world, must conform.
Bertrand Russell, The Study of Mathematics [1902]. Mathematician, philosopher, peace activist. Nobel Prize in Literature 1950.

It Ain't Fair

For a voting system (which ranks all candidates) to be *fair* it should *at least* have the following properties:

1. *No dictator*: No single voter always prevails.
2. *Respect unanimous choice*: If *every* voter ranks X ahead of Y then the voting results also do this.
3. *Ignore irrelevant preferences*: If some voters change the ranking of Z, but not the relative ranking of X and Y, then the election results also do not.

Theorem (Arrow's Impossibility Theorem)

No voting system is fair in the above sense.

Kenneth Arrow, mathematician, economist, Nobel Prize in Economics 1972

Proof.

Blah, blah □

Maximising Satisfaction

Theorem (Aumann)

Bartering, or buying & selling under a suitable price system, both lead to the same maximally satisfying distributions of goods, if we assume an infinite population.

Robert Aumann, mathematician, Nobel Prize Economics 2005

Proof.

1. Note the "infinite" population. Sensible?
2. (An aside for MATH 3320 students) The proof involves "set-valued measures"
3. Proof involves assigns sets of maximally satisfying commodity bundles to each consumer group. □

(Student: Nemanja Antic did masters thesis on this)

Fractals and the Money Markets

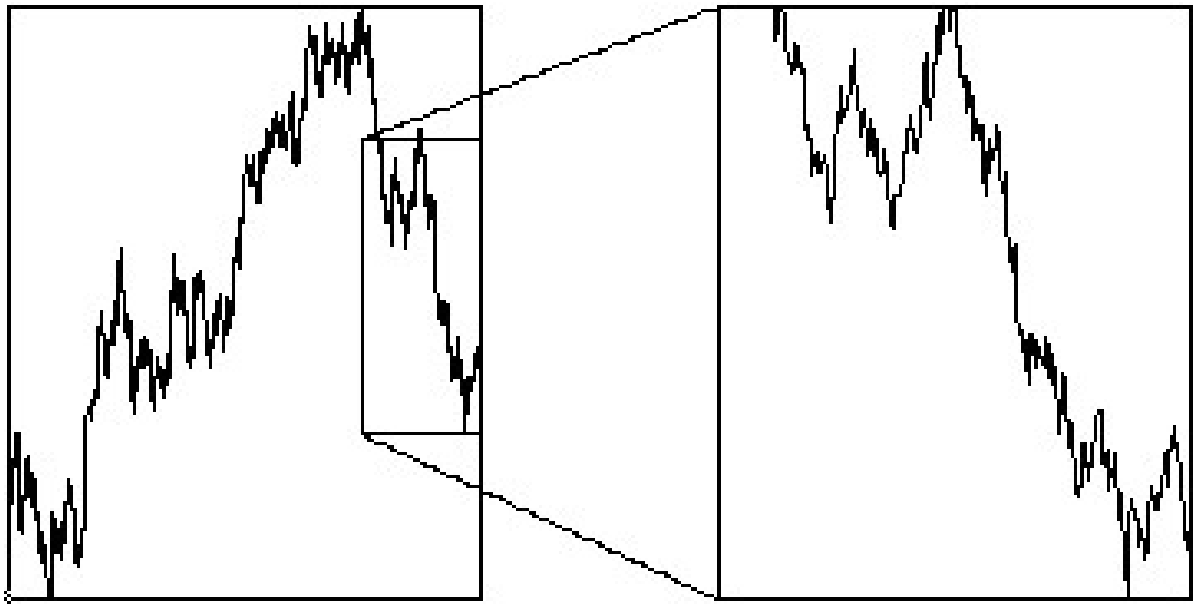


Figure: Zooming in on short term fluctuations on the money markets

Example of statistical self-similarity in a fractal

Fractals and Disaggregated materials

Figure: Zooming in on the coastline

Another example of statistical self-similarity in a fractal

Sierpinski Gasket

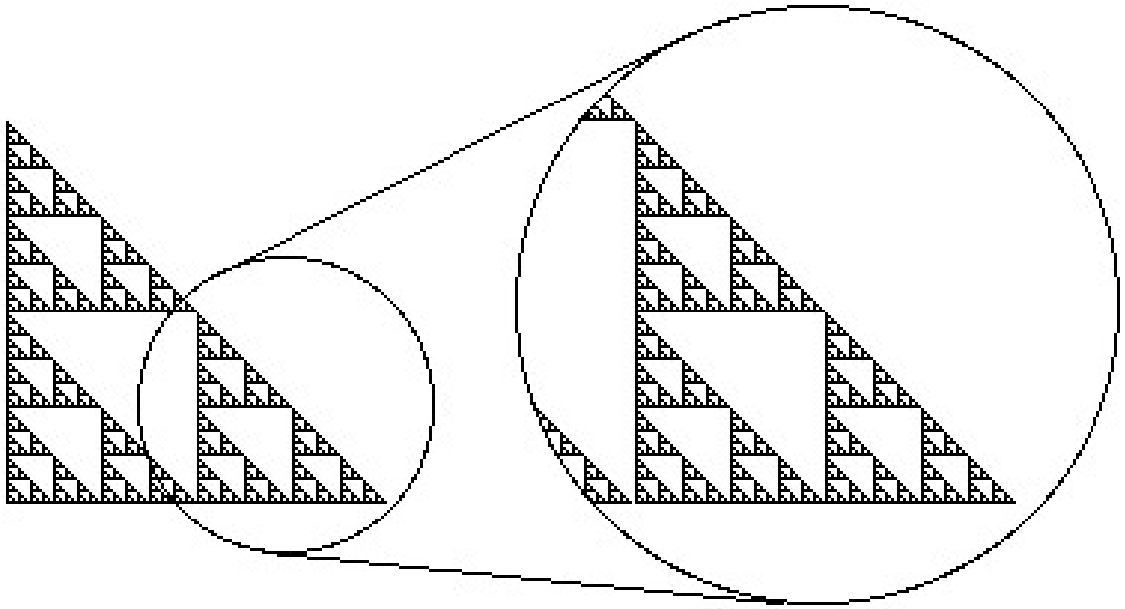


Figure: Zooming in on the Sierpinski Gasket

A useful mathematical fractal model.

Theorem (JH)

One can reconstruct fractals just from a knowledge of a small number of zoom maps (and hence deduce their properties).

Mandelbrot Set

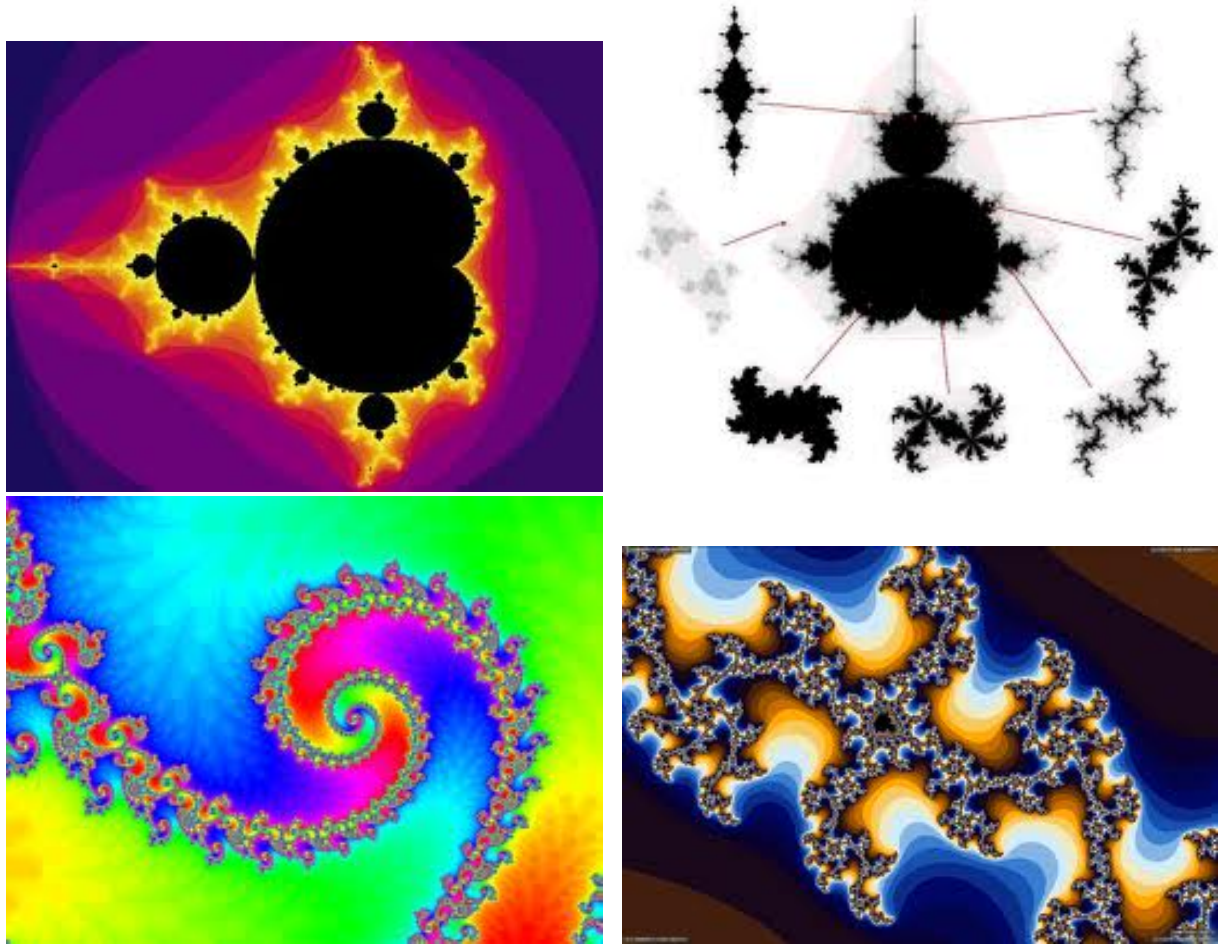


Figure: Mandelbrot Set

Factors and Cryptography

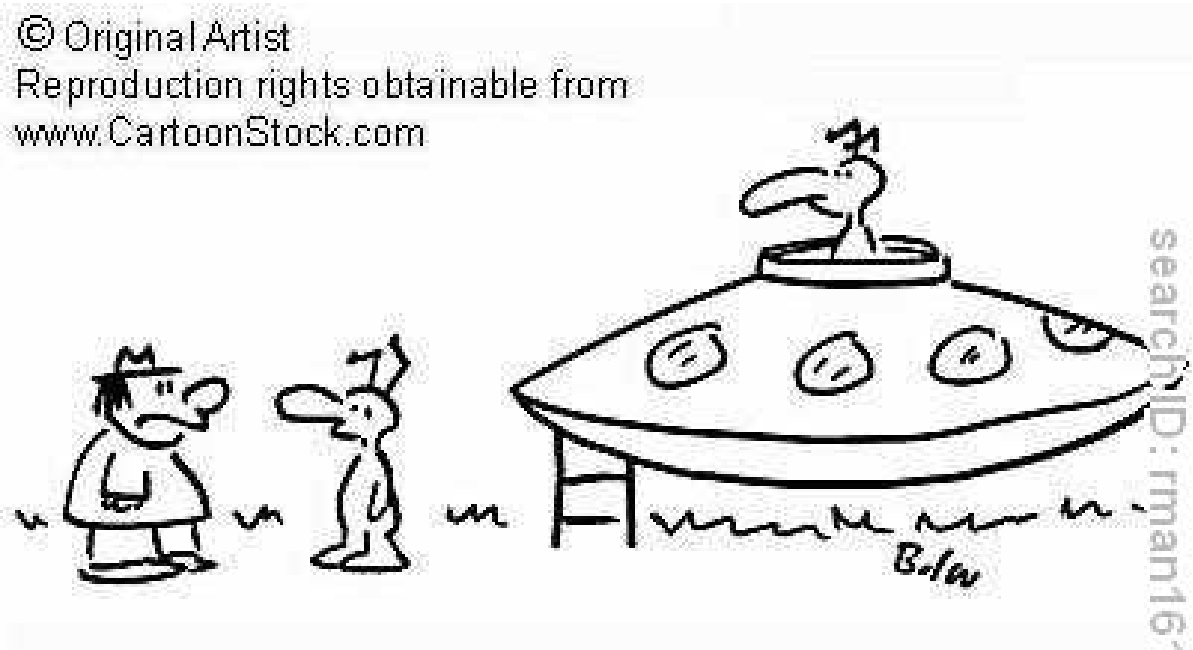
p=
836530018326471739718456981244510066629365454667349982776558965767174365745391729836879548878795879098374560987666398764099376398998764589702527386399
is a (very large) prime number, and so is

q=
943987627897384756895479800200710098812730100111111110586162605018856757411563456705614547510406538564747747846501601040 501476671045767176585018950251

Their product is

p×q =7896739876649618559910599579876168622125248048359272533123170700032841258320086508129154704201043151287918512598462553187927258900664171743292196973884356836180150868084489142979565014410035719974940343639550777859972441869692937933616094114878260225579841780591282181231663439807863351767216350361 49

Google



We have come to meet the Supreme All Knowing Being, the One you call "GOOGLE"

Searching

Google indexes and searches about 15,000 million web pages.
A search and ranking takes 1/10th - 3/10th of a second.

Method: ranking is based on first year linear algebra.

Google Founders:
Sergey Brin, mathematics & computer science (on leave from Stanford PhD program)
Larry Page, engineering & computer science (on leave from Stanford PhD program)

The Human Condition

What is at the core of being human ?
of being internally conscious of ourselves as free agents?
of internally experiencing sensations, emotions & feelings?

Free Will 1

Theorem (Conway, Kochen, 2005)

If there exist experimenters with some small amount of free will, then elementary particles also have some free will.

"Free will" $\Longrightarrow$ behaviour not completely determined by previous history of universe (at time and place of experiment)

Free Will 2

AXIOMS (i.e. Assumptions for Theorem)

1. FIN: Information travels no faster than the speed of light
2. SPIN: Some atoms (Deuterium) always have Spin 0 or 1; in 3 perpendicular directions give 101 in some order.

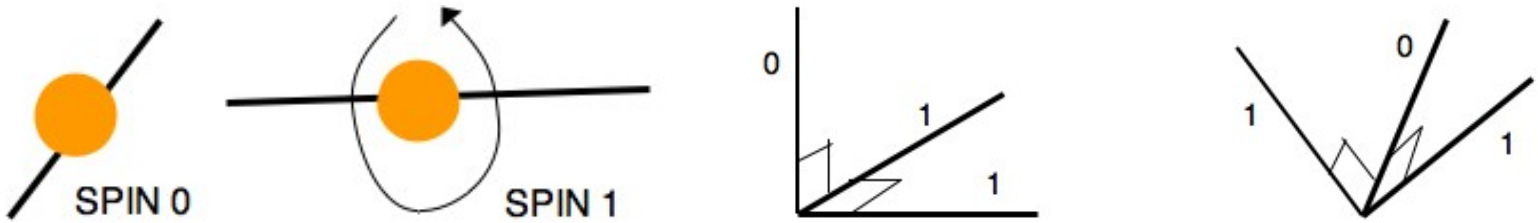


Figure: SPIN axiom

3. TWIN: Twinned atoms of this type, measured far apart always, have the same Spin.

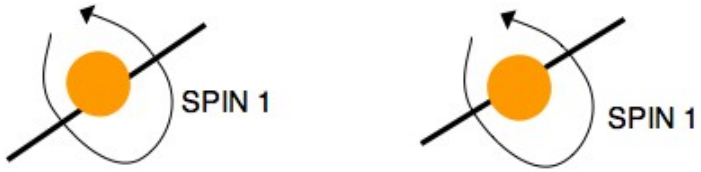


Figure: TWIN axiom

Free Will 3

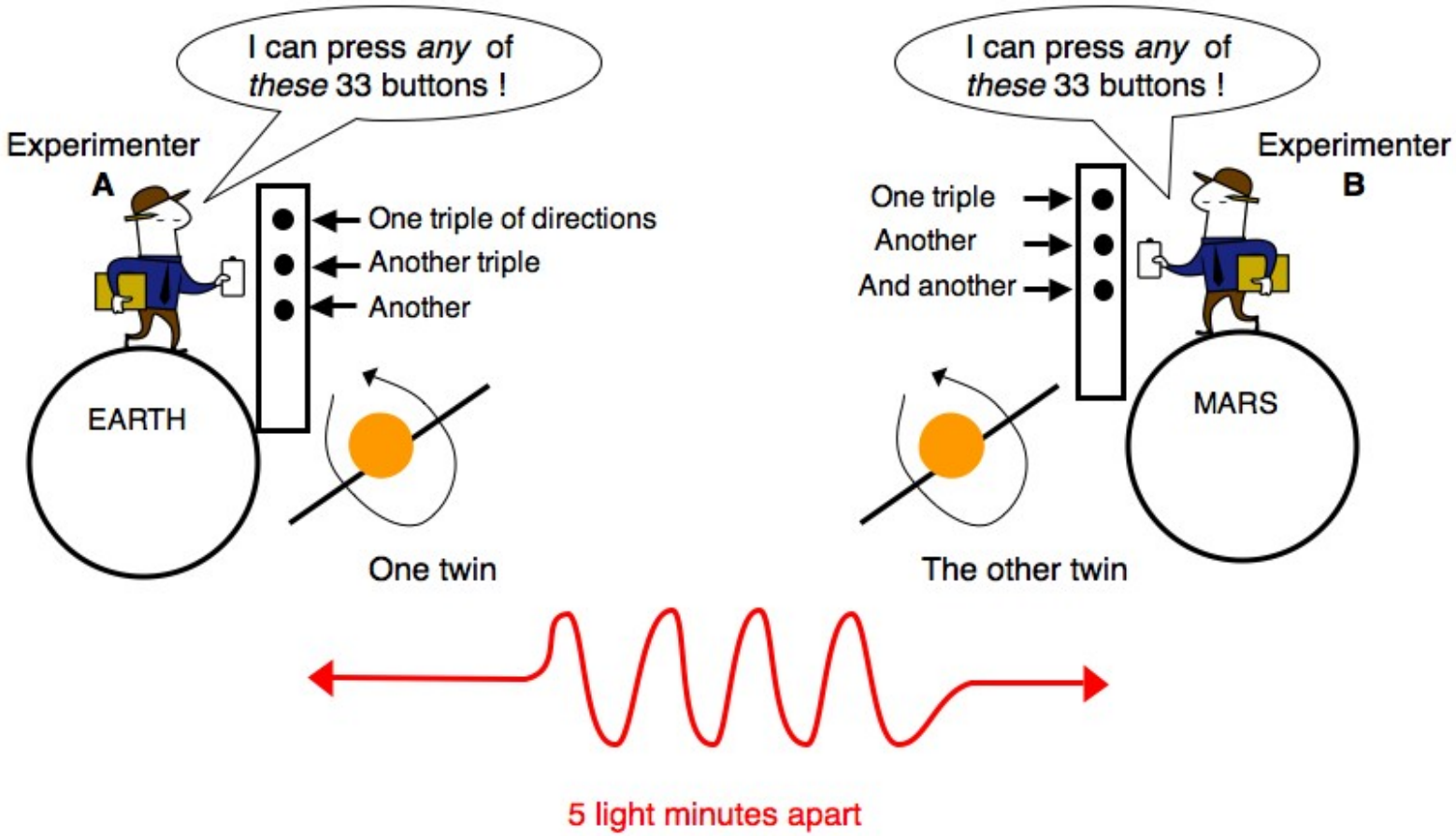


Figure: The thought experiment

Free Will 4

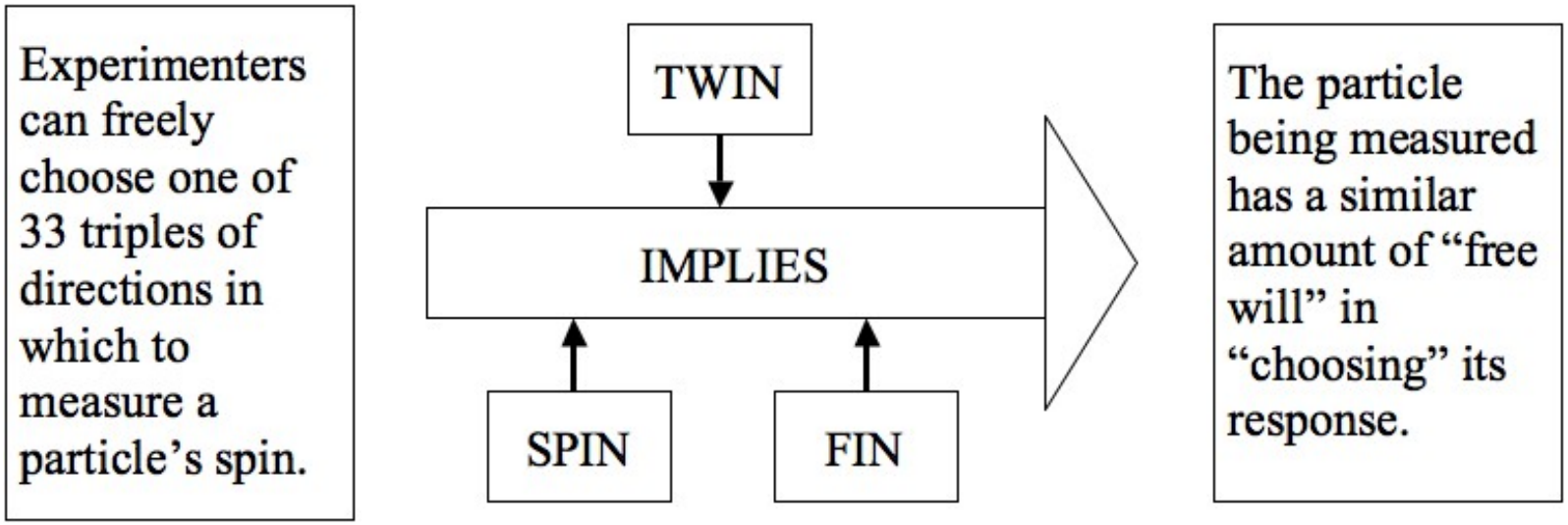


Figure: Conway-Kochen Theorem

Consciousness

There are axioms for properties of the integers, $+$ and $\times$.
We *believe* we cannot prove false statements.

By a tricky "diagonal" argument, it follows there is a statement S about the integers such that

1. S is *not provable* from the axioms;
2. but S is *true*.

(Sir) Roger Penrose, an eminent mathematical physicist, argues this idea can be extended to show we know mathematical and other truths that a computer cannot know.

His argument is controversial.

Conclusion:

The human mind will never be simulated by a computer.

Microtubules

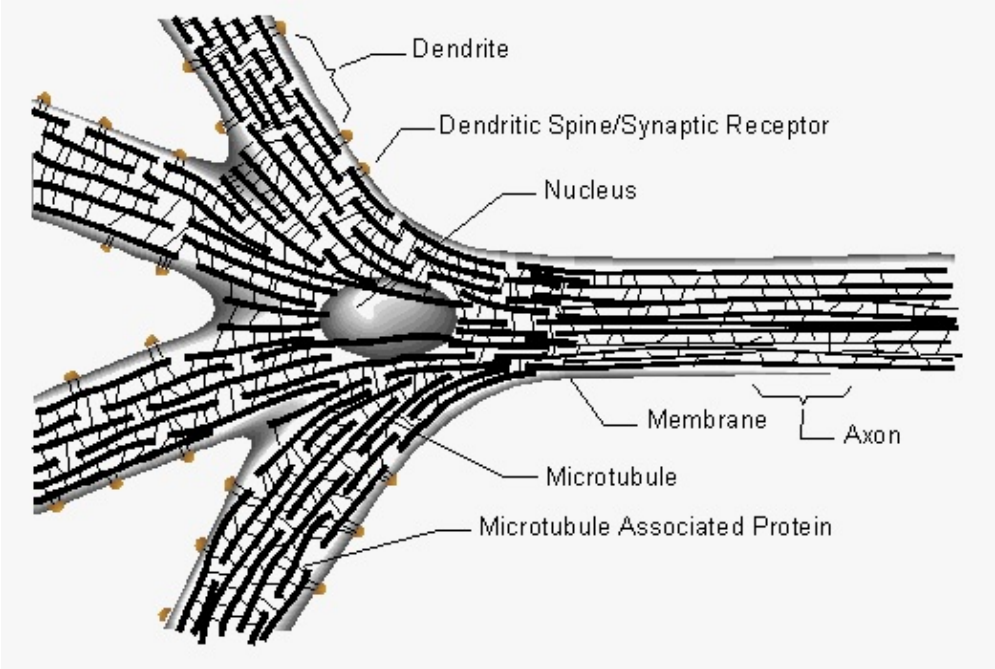


Figure: Central part of brain neuron

Penrose and Stuart Hameroff (medical researcher) conjecture consciousness is a result of quantum entaglement (twinning) in the microtubules of the neurons in the brain.
This is highly controversial.

Reflections on Teaching

1. Students Differ
2. ANU Secondary College Mathematics
3. School Teaching

Farewell

The ANU, your future careers, and any exams.

Course Notes (next week): `www.maths.anu.edu.au/~john/`