NEW SOUTH WALES

Department of Education

LEAVING CERTIFICATE EXAMINATION, 1962

Mathematics I

PASS PAPER

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Time allowed—Three hours

Candidates may attempt all questions.

Question 1 carries 30 marks and the other questions 10 marks each.

Except in Question 1, marks will not be awarded to answers where the work is not shown.

Marks will be deducted for careless or badly arranged work.

Mathematical Tables and Squared Paper will be provided.

Slide rules may not be used.

Answers to the two Parts of this Paper are to be returned in *separate books* marked A or B.

Part A

1. (i) Use logarithms to calculate $\sqrt[3]{1{\cdot}35 \times 0{\cdot}47}$ correct to two decimal places.

   (ii) If $A = 5{\cdot}4 \times 10^3$ and $B = 1{\cdot}5 \times 10^{-6}$, calculate, without using logarithms, the value of $\sqrt{\frac{A}{B}}$, giving your answer in the form $C \times 10^n$, where C and n are integers.

   (iii) X varies directly as the square of s and inversely as r. When $s = 6$ and $r = 3$, $X = 48$. Find the value of X when $s = 10$ and $r = 5$.

(iv) If a is a positive number and $\log_2 a = x$, write down the values of

(a) $\log_4 a$;
(b) $\log_a 4$.

(v) If S is the sum of the series
$$2 + 4 + 6 + \ldots + 2n$$
and T is the sum of the series
$$2 + 4 + 6 + \ldots + (2n - 2),$$
express $S^2 - T^2$ in its simplest form.

(vi) Write out the terms in the expansion, in powers of x, of the expression
$$\frac{1 + x}{(1 - x)^2},$$
as far as the term in x^3.

(vii) Find the values of the constants A, B, C and D, which are independent of n, such that
$$n^3 + n + 1 = An(n - 1)(n - 2) + Bn(n - 1) + Cn + D.$$

(viii) A particle moves in a straight line from rest at a fixed point O; its acceleration at time t seconds is $6t - 18$ feet per second per second. Find, at the time $t = 2$ seconds

(a) the velocity of the particle ;
(b) the distance of the particle from O.

(ix) Given that
$$\frac{dy}{dx} = 2x - 3,$$
and that the minimum value of y is $-\frac{1}{4}$, find y in terms of x.

(x) What is the area enclosed between the curve $y = 2x(3 - x)$ and the x-axis ?

2. (i) Differentiate

(a) $\dfrac{1}{x^2 - x + 1}$;

(b) $\tan^2\left(\dfrac{2x + 1}{3}\right)$.

(ii) Evaluate (expressing your answer in terms of surds and π where these quantities occur)

(a) $\displaystyle\int_1^2 \frac{1 + x}{\sqrt{x}}\, dx$;

(b) $\displaystyle\int_{\pi/6}^{\pi/3} \tan^2\theta \, d\theta$.

3. (i) Show that for all real values of a, b and h, the equation
$$(x - a)(x - b) - h^2 = 0$$
always has real roots. What is the condition for these roots to have the same sign?

(ii) Solve the equation
$$5^{2x} = 120 \times (0{\cdot}25)^{3x},$$
giving the solution correct to three decimal places.

Part B

4. (i) If
$$3y^2 = 2x^3 + x^2$$
find the value of $\dfrac{dy}{dx}$ when $x = 1$ and $y = 1$. Hence find the equation of the tangent at the point (1,1) to the curve represented by this equation. Show that this tangent cuts the curve again at a point at which $x = \frac{1}{6}$.

(ii) Prove that
$$\frac{1}{\sqrt{(x + h)} + \sqrt{x}} = \frac{\sqrt{(x + h)} - \sqrt{x}}{h}$$
and use this result to deduce the derivative of $\sqrt{x}$.

5. Show that the curves $y = 2 - x^2$ and $y = x^3$ intersect at the point (1,1) and at no other point. Sketch the two curves on the same diagram, over the range $-2 \leqslant x \leqslant 2$. (Graph paper need not be used.)

If O is the origin, P the point (1,1) and Q the point where the curve $y = 2 - x^2$ meets the positive x-axis, calculate to two decimal places the area of the region bounded by OQ and the arcs OP, PQ of the two curves.

6. (i) Draw a neat graph, to what you consider a suitable scale, of the function 3^x for values of x between $-1\cdot5$ and $+1\cdot5$. By drawing another appropriate graph, referred to the same axes, solve as accurately as possible the equation

$$3^x + 2x = 3.$$

Check your answer from the tables.

(ii) Find the numerical value of the term independent of x in the expansion of $\left(2x^2 - \frac{1}{2x}\right)^{12}$.

7. Prove that the volume of a spherical cap which is cut off from a sphere of radius a by a plane distant $a - h$ from the centre of the sphere is $\pi h^2(a - \frac{1}{3}h)$.

Water is poured into an upright hemispherical bowl of radius 5 inches, at the rate of 22 cubic inches per second. Taking π as $\frac{22}{7}$, calculate the rate at which the depth of the water is increasing when this depth is 3 inches.

8. (i) The n-th term of the series

$$3 + \tfrac{4}{3} + \tfrac{11}{18} + \ldots$$

is given by the formula $A(\frac{1}{3})^n + B(\frac{1}{2})^n$. Find the values of the constants A and B and thence obtain the fourth term of the series.

Find the sum of this series

(a) to n terms;
(b) to infinity.

(ii) Nine cards are marked, each with one of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9. The cards are shuffled and five are selected, one at a time. Calculate the probabilities

(a) that the card marked "9" is not drawn;
(b) that the digits on the cards drawn are in increasing order of magnitude.