Research

John Hutchinson

Contents

1 Publications

I was fortunate in having the opportunity to do research in a number of different areas of mathematics. Nowadays, with the pressure to continuously publish, it would be much more difficult if not impossible to have the luxury of exploring new fields.

1.1 Fractals and Stochastics

A lucky break came with [Hut81], it languished for a few years but then generated a lot of activity. On sabbatical in 1978 in Princeton, and at the suggestion of Fred Almgren, I undertook the project of putting some of the work on fractals by Benoit Mandelbrot into a unified mathematical framework. There has been a lot of subsequent work by others and each year since 2000 (when records were first kept), according to the American Mathematical Society this paper has been in the top 100 most cited articles in the mathematical literature, and some years in the top 10. Michael Barnsley and John Elton later took a different appoach via "the chaos game" and Michael subsequently created a company and successfully developed software and hardware to extend these ideas to image compression.

The papers [HR98, HR00a] with Ludger Rüschendorf develop a random version of this approach. In [BHS03, BHS05, BHS08, BHS12] with Michael and Örjan Stenflo the notion of V-variable fractals is developed. These are random fractals which in some sense are intermediate between homogeneous random fractals and standard random (i.e. recursive) fractals. The unpublished [BHS03] is an informal user-friendly inroduction. [FHH17] with Uta Freiberg and Ben Hambly gives a detailed analysis of the spectral properties of V-variable fractals.

[FHH17] U. Freiberg, B. M. Hambly, and John E. Hutchinson, *Spectral asymptotics for V-variable Sierpinski gaskets*, Ann. Inst. Henri Poincaré Probab. Stat. **53** (2017), no. 4, 2162–2213.

[BHS12] Michael Barnsley, John E. Hutchinson, and Örjan Stenflo, *V-variable fractals: dimension results*, Forum Math. **24** (2012), no. 3, 445–470.

[BHS08] ———, *V-variable fractals: fractals with partial self similarity*, Adv. Math. **218** (2008), no. 6, 2051–2088.

[BH06] Michael Barnsley and John E. Hutchinson, *New methods in fractal imaging*, International Conference on Computer Graphics, Imaging and Visualisation (CGIV'06), IEEE, 2006.

[BHS05] Michael Barnsley, John E. Hutchinson, and Örjan Stenflo, *A fractal valued random iteration algorithm and fractal hierarchy*, Fractals **13** (2005), no. 2, 111–146.

[BHS03] ———, *V-variable fractals and superfractals* (2003), https://arxiv.org/abs/math/0312314.

[HR00a] John E. Hutchinson and Ludger Rüschendorf, *Random fractals and probability metrics*, Adv. in Appl. Probab. **32** (2000), no. 4, 925–947.

[HR00b] ———, *Selfsimilar fractals and selfsimilar random fractals*, Fractal geometry and stochastics, II (1998), Progr. Probab., vol. 46, Birkhäuser, Basel, 2000, pp. 109–123.

[Hut00] John E. Hutchinson, *Deterministic and random fractals*, Complex systems, Cambridge Univ. Press, Cambridge, 2000, pp. 127–166.

[HR98] John E. Hutchinson and Ludger Rüschendorf, *Random fractal measures via the contraction method*, Indiana Univ. Math. J. **47** (1998), no. 2, 471–487.

[Hut95] John E. Hutchinson, *Fractals: a mathematical framework*, Complex. Int. **2** (1995), 14.

[Hut91] ———, *Generating fractals*, Chaos & order (Canberra, 1990), World Sci. Publ., 1991, pp. 76–81.

[Hut81] ———, *Fractals and self-similarity*, Indiana Univ. Math. J. **30** (1981), no. 5, 713–747.

1.2 Geometric Measure Theory, Analytic Methods for Geometric Problems

The landmark paper *Normal and Integral Currents* by Federer and Fleming was a major breakthrough in the study of gometric variational problems. In 1975 at the ANU, Neil Trudinger encouraged a number of us to run a seminar series on Federer's book *Geometric Measure Theory*. At around the same time Enrico Giusti visited the ANU and gave a series of lectures on a related approach due to De Giorgi for hypersurfaces.

[Hut81] answers a question of De Giorgi, pointed out to me by Giusti, concerning the equivalence of two measures for hypersurfaces, and gives a negative answer with the proof motivated by fluid flow around a Cantor type set in the plane. [Hut86] develops a suggestion of Leon Simon and introduces a generalised notion of curvature for objects potentially with singularities. Related regularity issues are addressed in [Hut86, Hut87, Hut90]. The nonuniqueness of tangent cones in [HM86] with Michael Meier answers a small question of Allard. The paper [FH97] with Yi Fang introduces a number of novel ideas, but unfortunately has slipped under the radar. [HT00] with Yoshi Tonegawa studies phase transition interfaces in the geometric measure theory setting.

[HT00] John E. Hutchinson and Yoshihiro Tonegawa, *Convergence of phase interfaces in the van der Waals-Cahn-Hilliard theory*, Calc. Var. Partial Differential Equations **10** (2000), no. 1, 49–84.

[FH97] Yi Fang and John E. Hutchinson, *Hessian energy and optimal parametrisations of surfaces*, Math. Z. **225** (1997), no. 3, 507–535.

[Hut90] John E. Hutchinson, *Poincaré-Sobolev and related inequalities for submanifolds of* $\mathbf{R}^N$, Pacific J. Math. **145** (1990), no. 1, 59–69.

[Hut87] ———, *Some regularity theory for curvature varifolds*, Miniconference on geometry and partial differential equations, 2 (Canberra, 1986), Proc. Centre Math. Anal. Austral. Nat. Univ., vol. 12, Austral. Nat. Univ., Canberra, 1987, pp. 60–66.

[Hut86] ———, *$C^{1,\alpha}$ multiple function regularity and tangent cone behaviour for varifolds with second fundamental form in L^p*, Geometric measure theory and the calculus of variations (Arcata, Calif., 1984), Proc. Sympos. Pure Math., vol. 44, Amer. Math. Soc., Providence, RI, 1986, pp. 281–306.

[HM86] John E. Hutchinson and Michael Meier, *A remark on the nonuniqueness of tangent cones*, Proc. Amer. Math. Soc. **97** (1986), no. 1, 184–185.

[Hut86] John E. Hutchinson, *Second fundamental form for varifolds and the existence of surfaces minimising curvature*, Indiana Univ. Math. J. **35** (1986), no. 1, 45–71.

[Hut84] ———, *Minimising curvature—a higher-dimensional analogue of the Plateau problem*, Miniconference on nonlinear analysis (Canberra, 1984), Proc. Centre Math. Anal. Austral. Nat. Univ., vol. 8, 1984, pp. 113–122.

[Hut81] ———, *On the relationship between Hausdorff measure and a measure of De Giorgi, Colombini and Piccinini*, Boll. Un. Mat. Ital. B (5) **18** (1981), no. 2, 619–628.

1.3 Numerical Analysis for Geometric Problems

In the early 1990's, through the Centre for Mathematical Analysis [CMA] directed by Neil Trudinger and containing a very active research group led by Leon Simon, I met Gerd Dziuk. This led to a fruitful collaboration [DH94] ... [DH06] and [Hut91] involving theoretical and numerical work concerning geometric problems.

[DH06] Gerhard Dziuk and John E. Hutchinson, *Finite element approximations to surfaces of prescribed variable mean curvature*, Numer. Math. **102** (2006), no. 4, 611–648.

[DH99a] ———, *The discrete Plateau problem: convergence results*, Math. Comp. **68** (1999), no. 226, 519–546.

[DH99b] ———, *The discrete Plateau problem: algorithm and numerics*, Math. Comp. **68** (1999), no. 225, 1–23.

[DH98] ———, *Finite element approximations and the Dirichlet problem for surfaces of prescribed mean curvature*, Mathematical visualization (1997), Springer, Berlin, 1998, pp. 73–87.

[DH96] ———, *On the approximation of unstable parametric minimal surfaces*, Calc. Var. Partial Differential Equations **4** (1996), no. 1, 27–58.

[DH95] ———, *L^2 estimates for approximations to minimal surfaces*, Curvature flows and related topics (Levico, 1994), GAKUTO Internat. Ser. Math. Sci. Appl., vol. 5, 1995, pp. 67–82.

[DH94] ———, *A finite element method for the computation of parametric minimal surfaces*, Tatra Mt. Math. Publ. **4** (1994), 49–62. Equadiff 8 (Bratislava, 1993).

[Hut91] John E. Hutchinson, *Computing conformal maps and minimal surfaces*, Workshop on Theoretical and Numerical Aspects of Geometric Variational Problems (Canberra, 1990), Proc. Centre Math. Appl. Austral. Nat. Univ., vol. 26, Austral. Nat. Univ., Canberra, 1991, pp. 140–161.

1.4 Multivariable Variational Problems, Regularity and Singularities of Solutions

In the same CMA milieu as mentioned before, but a few years earlier, I met Nicola Fusco. This led to another very fruitful collaboration, with [FH85] ... [FH95] motivated initially by Giaquinta's book *Multiple Integrals in the Calculus of Variations and Nonlinear Elliptic Systems*. Separately, [AFH03] with Luigi Ambrosio and Nicola studies the dimension of the singular set for the Mumford-Shah functional used in image segmentation and pattern recognition.

[AFH03] Luigi Ambrosio, Nicola Fusco, and John E. Hutchinson, *Higher integrability of the gradient and dimension of the singular set for minimisers of the Mumford-Shah functional*, Calc. Var. Partial Differential Equations **16** (2003), no. 2, 187–215.

[FH95] Nicola Fusco and John E. Hutchinson, *A direct proof for lower semicontinuity of polyconvex functionals*, Manuscripta Math. **87** (1995), no. 1, 35–50.

[FH94] ———, *Partial regularity and everywhere continuity for a model problem from non-linear elasticity*, J. Austral. Math. Soc. Ser. A **57** (1994), no. 2, 158–169.

[FH91] ———, *Partial regularity in problems motivated by nonlinear elasticity*, SIAM J. Math. Anal. **22** (1991), no. 6, 1516–1551.

[FH89] ———, *Partial regularity for minimisers of certain functionals having nonquadratic growth*, Ann. Mat. Pura Appl. (4) **155** (1989), 1–24.

[FH86] ———, *Partial regularity for solutions of variational problems*, Miniconference on geometry and partial differential equations (Canberra, 1985), Proc. Centre Math. Anal. Austral. Nat. Univ., vol. 10, 1986, pp. 57–67.

[FH85] ———, *$C^{1,\alpha}$ partial regularity of functions minimising quasiconvex integrals*, Manuscripta Math. **54** (1985), no. 1-2, 121–143.

1.5 Mathematical Logic, Model Theory & Set Theory

After many enjoyable years as a graduate student at Stanford just south of San Francisco, but not fruitful by way of publishable mathematics, I changed supervisors to Harvey Friedman. Harvey at 18 was younger than any graduate student, and had just taken up a position at Stanford after his PhD from MIT. Following some reading suggestions of his, I came up with the result in [Hut76c] which gave a new class of extensions of models of set theory. Since I was slow but careful, when Harvey said he did not believe the result I knew it was an interesting one, and anyway the next day he said he did believe it. [Hut76a] uses these and other ideas to give a short and unified way of proving old and then

new results for infinitary logics. [Hut76a] classifies the order types of ordinals when seen from outside the model itself of set theory.

On returning to Australia after my PhD there was little in the way of activity in the field of mathematical logic and so I resolved to change fields. I was fortunate to obtain a position at the ANU and then took the opportunity to move into the general area of geometric measure theory in **1.2**.

[Hut76a] John E. Hutchinson, *Model theory via set theory*, Israel J. Math. **24** (1976), no. 3–4, 286–304.

[Hut76b] ———, *Order types of ordinals in models of set theory*, J. Symbolic Logic **41** (1976), no. 2, 489–502.

[Hut76c] ———, *Elementary extensions of countable models of set theory*, J. Symbolic Logic **41** (1976), no. 1, 139–145.

2 Grants

- *A Numerical and Theoretical Study of Models for the Evolution of Interfaces* with G. Huisken and S. Roberts, Faculties Research Fund, 1989, 90
- *A Numerical and Theoretical Study of Models for the Evolution of Interfaces* with G. Huisken and S. Roberts, 1989–91, Australian Research Council (ARC) large grant (A68830148)
- *Theoretical and Numerical Aspects of Geometric Variational Problems* with G. Dziuk and G. Huisken, 1990, a bilateral exchange workshop, DITAC and German Research Foundation
- *Nonlinear Analysis and Geometric Problems* with R. Bartnik, G. Huisken, N. Trudinger, J. Urbas, ARC large grant (A69031884), 1991-94 (rolled over)
- *An Investigation of Finite Element Approximations to Minimal Surfaces, H-surfaces, and to Systems of Partial Differential Equations; and of Theoretical and Computational Aspects of Nonlinear Elasticity Type Problems* ARC small grant F95078, 1995
- *A Mathematical Investigation of Curvatures and Various Energies Related to Geometric and Physical Problems* ARC large grant (A69602500), 1996-99 (rolled over)
- *Numerical Algorithms for investigating physical and geometric processes* 2001-2002, Australian-German Joint Research Co-operation Scheme
- *Theoretical and Computational Investigations into Nonlinear Geometric and Physical Variational Problems* ARC large grant (A001042272), 2001-2004 (rolled over)
- *A new generation of fractals: theory, computation, and applications particularly to digital imaging* with M. Barnsley, ARC large grant (0558974), 2005-2010 (rolled over)

3 Patent

Fractal image data and image generator with M. Barnsley, and Ö. Stenflo, through ANUTech

- Australian Provisional Patent, March 2003
- Shortlisted – Secrets of Australian IT awards 2003
- ACT Knowledge Fund Grant 2004 ($25,000)
- International Provisional patent, March 2004
- Publication of US20070040848A1, Febuary 2007
- Application Status: abandoned, May 2019

Unfortunately, this ended up in the category of a solution looking for a problem.
https://patentimages.storage.googleapis.com/e3/f2/e0/39201089ef487c/US20070040848A1.pdf

4 Links to Articles

Google Scholar
Mathscinet from ANU
Mathscinet with other login